

# Math 115

## Fall 2017

### Lecture 21



Class Quiz

1) Find  $(2x+3)^2$

$$= (2x+3)(2x+3) = 4x^2 + 6x + 6x + 9$$

$$= \boxed{4x^2 + 12x + 9}$$

2) Divide:  $\frac{12x^4 - 6x^2}{3x^2}$

$$= \frac{12x^4}{3x^2} - \frac{6x^2}{3x^2}$$

$$= \boxed{4x^2 - 2}$$

3) Divide:  $\frac{x^2 - 6x + 5}{x-2}$

$$\begin{array}{r} x \ -4 \\ x-2 \overline{) x^2 - 6x + 5} \\ \underline{-(x^2 - 2x)} \phantom{+5} \\ -4x + 5 \\ \underline{-(-4x + 8)} \\ -3 \end{array}$$

$$\begin{aligned} x \square &= x^2 & - (x^2 - 2x) \\ x \square &= -4x & - (-4x + 8) \end{aligned}$$

$$\boxed{x - 4 + \frac{-3}{x-2}}$$

Factor out the GCF

$$1) 4x^2 - 16x$$

$$= 4x(x - 4)$$

$$2) 10x^3 - 20x^2 + 50x$$

$$= 10x(x^2 - 2x + 5)$$

$$3) 3x(7x+2) - 5(7x+2)$$

$$= (7x+2)(3x-5)$$

$$4) x^2(2x-5) - 3x(2x-5) - 8(2x-5)$$

$$= (2x-5)(x^2 - 3x - 8)$$

Factor by grouping:

$$5) 5x^3 + 3x^2 + 10x + 6$$

$$= x^2(5x+3) + 2(5x+3) = (5x+3)(x^2+2)$$

$$6) 7x^3 - 2x^2 + 70x - 20$$

$$= x^2(7x-2) + 10(7x-2) = (7x-2)(x^2+10)$$

$$7) 12x^3 + 15x^2 - 4x - 5$$

$$= 3x^2(4x+5) - 1(4x+5) = (4x+5)(3x^2-1)$$

Factor

$$1) \quad x^2 + 4x + 3$$

Diagram: A V-shape with 1 and 3 on the left, 3 and 3 on the right, and 3 in the middle. Arrows point from 1 and 3 to 3, and from 3 and 3 to 3. Labels:  $P=3$ ,  $S=4$ .

$$= x^2 + x + 3x + 3$$

$$= x(x+1) + 3(x+1)$$

$$= \boxed{(x+1)(x+3)}$$

$$2) \quad x^2 - 2x - 24$$

Diagram: A V-shape with -6 and 4 on the left, -24 and -24 on the right, and -24 in the middle. Arrows point from -6 and 4 to -24, and from -24 and -24 to -24. Labels:  $P=-24$ ,  $S=-2$ .

$$= x^2 - 6x + 4x - 24$$

$$= x(x-6) + 4(x-6)$$

$$= \boxed{(x-6)(x+4)}$$

$$3) \quad x^2 - 8x + 15$$

Diagram: A V-shape with -5 and -3 on the left, 15 and 15 on the right, and 15 in the middle. Arrows point from -5 and -3 to 15, and from 15 and 15 to 15. Labels:  $P=15$ ,  $S=-8$ .

$$= x^2 - 5x - 3x + 15$$

$$= x(x-5) - 3(x-5)$$

$$= \boxed{(x-5)(x-3)}$$

$$4) \quad x^2 - 20x + 100$$

Diagram: A V-shape with -10 and -10 on the left, 100 and 100 on the right, and 100 in the middle. Arrows point from -10 and -10 to 100, and from 100 and 100 to 100. Labels:  $P=100$ ,  $S=-20$ .

$$= x^2 - 10x - 10x + 100$$

$$= x(x-10) - 10(x-10)$$

$$= \boxed{(x-10)^2}$$

Factor Completely:

$$1) \quad 2x^2 + 7x - 15$$

Diagram: A V-shape with -3 and 10 on the left, -30 and -30 on the right, and -30 in the middle. Arrows point from -3 and 10 to -30, and from -30 and -30 to -30. Labels:  $P=-30$ ,  $S=7$ .

$$= 2x^2 - 3x + 10x - 15$$

$$= x(2x-3) + 5(2x-3)$$

$$= \boxed{(2x-3)(x+5)}$$

$$2) \quad 6x^2 - 5x - 6$$

Diagram: A V-shape with 4 and -9 on the left, -36 and -36 on the right, and -36 in the middle. Arrows point from 4 and -9 to -36, and from -36 and -36 to -36. Labels:  $P=-36$ ,  $S=-5$ .

$$= 6x^2 + 4x - 9x - 6$$

$$= 2x(3x+2) - 3(3x+2)$$

$$= \boxed{(3x+2)(2x-3)}$$

Factor Completely:

$$\begin{aligned}
 1) \quad & 10x^2 + 23x + 12 \\
 & \begin{array}{c} \text{8} \times \text{15} \quad \text{P}=120 \\ \quad \quad \quad \text{S}=23 \end{array} \\
 & \quad \quad \quad \text{120} \\
 & = 10x^2 + 8x + 15x + 12 \\
 & = 2x(5x+4) + 3(5x+4) \\
 & = \boxed{(5x+4)(2x+3)}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & 6x^2 - 23x + 20 \\
 & \begin{array}{c} -15 \times -8 \quad \text{P}=120 \\ \quad \quad \quad \text{S}=-23 \end{array} \\
 & \quad \quad \quad \text{120} \\
 & = 6x^2 - 15x - 8x + 20 \\
 & = 3x(2x-5) - 4(2x-5) \\
 & = \boxed{(2x-5)(3x-4)}
 \end{aligned}$$

Use  $A^2+B^2$ ,  $A^2-B^2$ ,  $A^3+B^3$ , and  $A^3-B^3$  to factor

$$\begin{aligned}
 1) \quad & 36x^2 - 49 \\
 & = (6x)^2 - (7)^2 \\
 & \quad \quad \quad A^2 - B^2 \\
 & = \boxed{(6x+7)(6x-7)}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & 25x^2 + 36 \\
 & = (5x)^2 + (6)^2 \\
 & \quad \quad \quad A^2 + B^2 \quad \text{Prime}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & 125x^3 + 8 \\
 & = (5x)^3 + (2)^3 \\
 & \quad \quad \quad A^3 + B^3 \\
 & = (5x+2)(25x^2 - 10x + 4)
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & 27x^3 - 64 \\
 & = (3x)^3 - (4)^3 \\
 & \quad \quad \quad A^3 - B^3 \\
 & = (3x-4)(9x^2 + 12x + 16)
 \end{aligned}$$

Use  $A^2 + 2AB + B^2 = (A+B)^2$  or  
 $A^2 - 2AB + B^2 = (A-B)^2$  to factor

$$1) 121x^2 - 44x + 4 = (11x - 2)^2$$

$$2 \cdot 11x \cdot 2 \checkmark$$

$$2) 49x^2 + 168xy + 144y^2 = (7x + 12y)^2$$

$$2 \cdot 7x \cdot 12y \checkmark$$

Factor Completely:

$$1) 20x^3y^2 - 15x^2y^3$$

$$= 5x^2y^2(4x - 3y)$$

$$2) 2x^2 - 200$$

$$= 2(x^2 - 100)$$

$$= 2(x^2 - 10^2)$$

$A^2 - B^2$

$$= 2(x + 10)(x - 10)$$

$$3) 75x^2 - 27y^2$$

$$= 3(25x^2 - 9y^2)$$

$$= 3[(5x)^2 - (3y)^2]$$

$$= 3(5x + 3y)(5x - 3y)$$

$$4) x^2 - 3x + 9$$

1, 9

3, 3

9

P=9

S=-3

Prime

$$5) x^2 - 16 + 6x$$

$$= x^2 + 6x - 16$$

$$= (x+8)(x-2)$$

1, 12  
2, 36  
3, 24  
⋮  
8, 9

$$6) 12x^2 + 34x + 24$$

$$= 2(6x^2 + 17x + 12)$$

72      P=72  
S=17

$$= 2(6x^2 + 8x + 9x + 12)$$

$$= 2[2x(3x+4) + 3(3x+4)]$$

$$= 2(3x+4)(2x+3)$$

$$7) 12x^3 - 24x^2 + 4x$$

$$= 4x(3x^2 - 6x + 1)$$

$$8) 4x^3 + 20x^2 - 56x$$

$$= 4x(x^2 + 5x - 14) = 4x(x+7)(x-2)$$

$$9) 2x - 5 + 3x^2$$

$$= 3x^2 + 2x - 5$$

$$= (3x+5)(x-1)$$

+5x  
-3x

$$10) x^3 + 7x^2 - 4x - 28$$

$$= x^2(x+7) - 4(x+7)$$

$$= (x+7)(x^2 - 4)$$

$A^2 - B^2$

$$= (x+7)(x+2)(x-2)$$

11)  $3x^2 - 5x + 1$   
 Factors:  $1, 3$  and  $3, -5$   
 $P=3$   
 $S=-5$

Prime

12)  $x^2y^2 - 9x^2 + 3y^2 - 27$

$$= x^2(y^2 - 9) + 3(y^2 - 9)$$

$$= (y^2 - 9)(x^2 + 3)$$

$A^2 - B^2$

$$= (y + 3)(y - 3)(x^2 + 3)$$

13)  $27x^4y - 125xy^4$

Hint:  
GCF,  $A^3 - B^3$

$$= xy[27x^3 - 125y^3]$$

$$= xy[(3x)^3 - (5y)^3]$$

$$= xy(3x - 5y)(9x^2 + 15xy + 25y^2)$$

$$14) 5x^4 - 115x^3 + 600x^2$$

Hint:  
GCF

$$= 5x^2(x^2 - 23x + 120)$$

$$= 5x^2(x - 15)(x - 8)$$